Starter Question – 2018 P1

13 A vehicle, which begins at rest at point *P*, is travelling in a straight line.

For the first 4 seconds the vehicle moves with a constant acceleration of $0.75\,\text{m}\,\text{s}^{-2}$

For the next 5 seconds the vehicle moves with a constant acceleration of $-1.2\,\text{m}\,\text{s}^{-2}$

The vehicle then immediately stops accelerating, and travels a further 33 m at constant speed.

13 (a) Draw a velocity–time graph for this journey on the grid below.

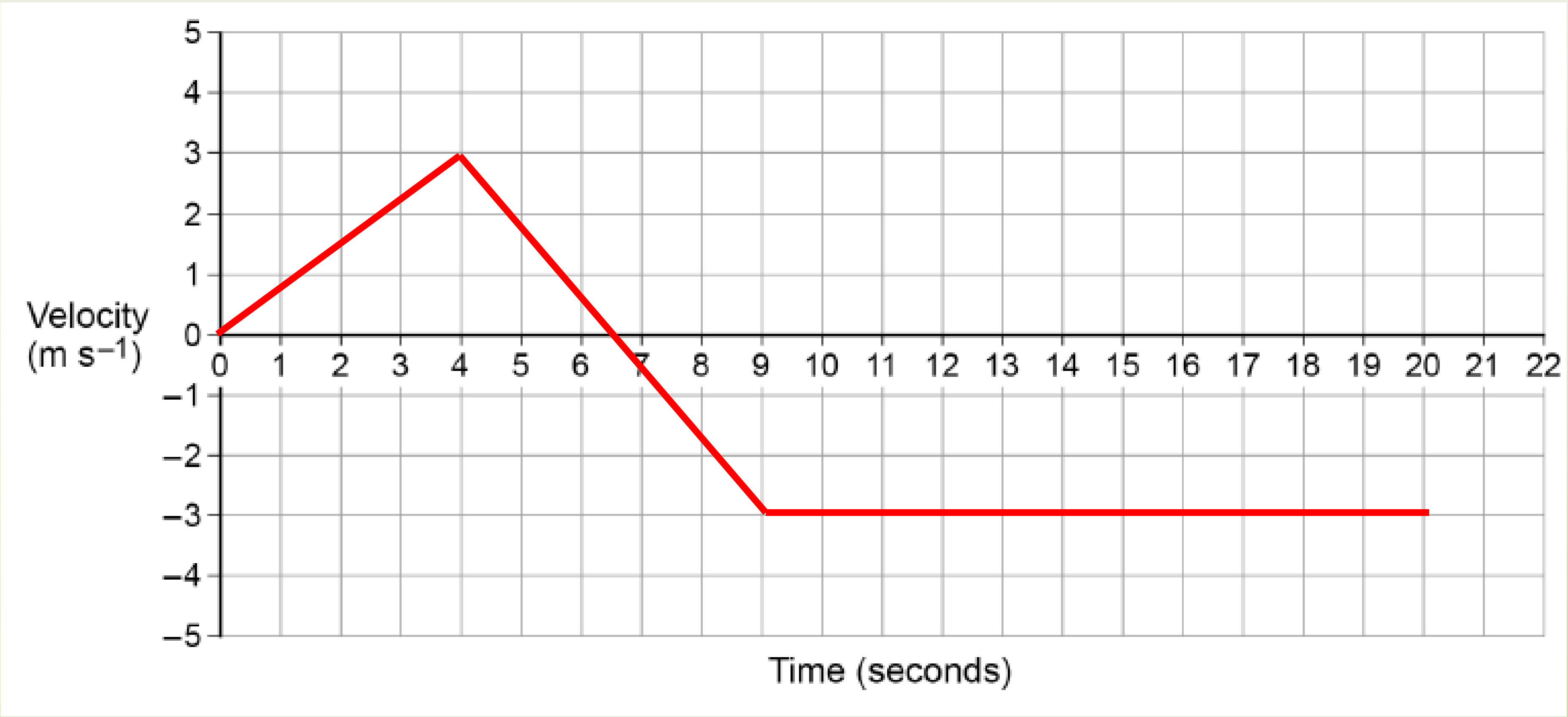
[3 marks]

13 (b) Find the distance of the car from *P* after 20 seconds.

[3 marks]

Starter Question

ne 2018 Paper 1 Q13a



Q13b = 27m

Q	Marking Instructions	AO	Marks	Typical Solution
13(a)	Draws first stage of graph correctly.	AO3.3	B1	Line connecting (0, 0) to (4, 3).
	Draws second stage correctly Line moving down by 6 ms^{-1} in 5s	AO3.3	B1F	Line connecting (4, 3) to (9, -3).
	Deduces final stage correctly Horizontal line of length $33/v$ where v is the speed of the vehicle at the end of the second stage.	AO2.2a	B1F	Line connecting (9, -3) to (20, -3).
13(b)	Finds at least one non-rectangular area. As typical solution **Or** t between 0 & 6.5: Area of triangle = 3 x 6.5 ÷ 2 = 9.75 m (forward) t between 6.5 & 20: Area of trapezium = 0.5(13.5 + 11) x (-3) = -36.75 m (backwards)	AO1.1a	M1	t between 0 & 4: Area of triangle = 3 x 4 ÷ 2 = 6 m (forward) t between 4 & 6.5: Area of triangle = 3 x 2.5 ÷ 2 = 3.75 m (forward) t between 6.5 & 9: Area of triangle = 2.5 x -3 ÷ 2 = -3.75 m (backwards) t between 9 and 20: Area of rect. = -3 x 11 = -33 m (backwards)
	Subtracts *their areas* below axis from areas above axis. OE	AO1.1a	M1	Displacement from O is 9.75 – 36.75 = –27 m
	Calculates distance from P correctly. CAO. Units not required	AO3.2a	A1	Distance is 27m
	Total		**6**	

Q

Kinematics

Q3

Understand, use and derive the formulae for constant acceleration for motion in a straight line; extend to 2 dimensions using vectors.

Assessed at AS and A-level

Teaching guidance

Students should be able to:

- recall and use the following formulae:

$$v = u + at$$

$$s = vt - \frac{1}{2}at^2$$

$$s = \frac{1}{2}(u + v)t$$

$$v^2 = u^2 + 2as$$

$$s = ut + \frac{1}{2}at^2$$

Note: the less fashionable $s = vt - \frac{1}{2}at^2$ is not essential.

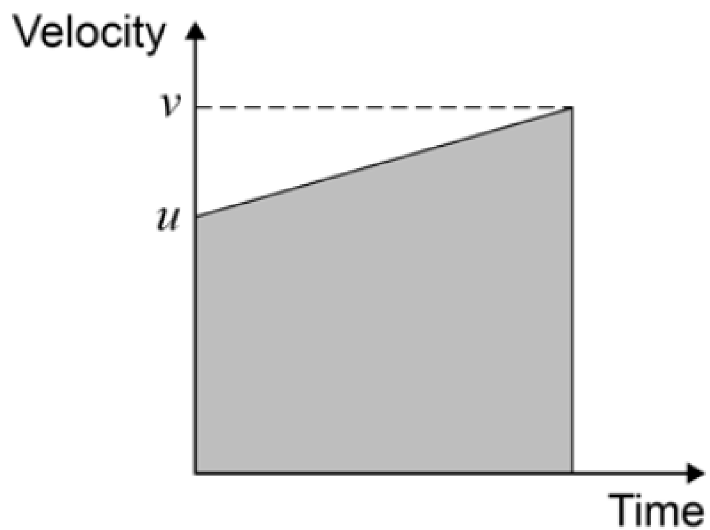
Q Kinematics

Q3

Understand, use and derive the formulae for constant acceleration for motion in a straight line; extend to 2 dimensions using vectors.

- derive the above formulae starting from given assumptions.

This may include starting from a graph similar to the one below.



Key
s = displacement
u = initial velocity
v = final velocity
a = acceleration
t = time

For example, the shaded area gives the displacement, s, and can be found as the area of a trapezium:

$$s = \frac{1}{2}(u+v)t$$

7.3 Equations of motion for constant acceleration

All motion involves the following quantities:

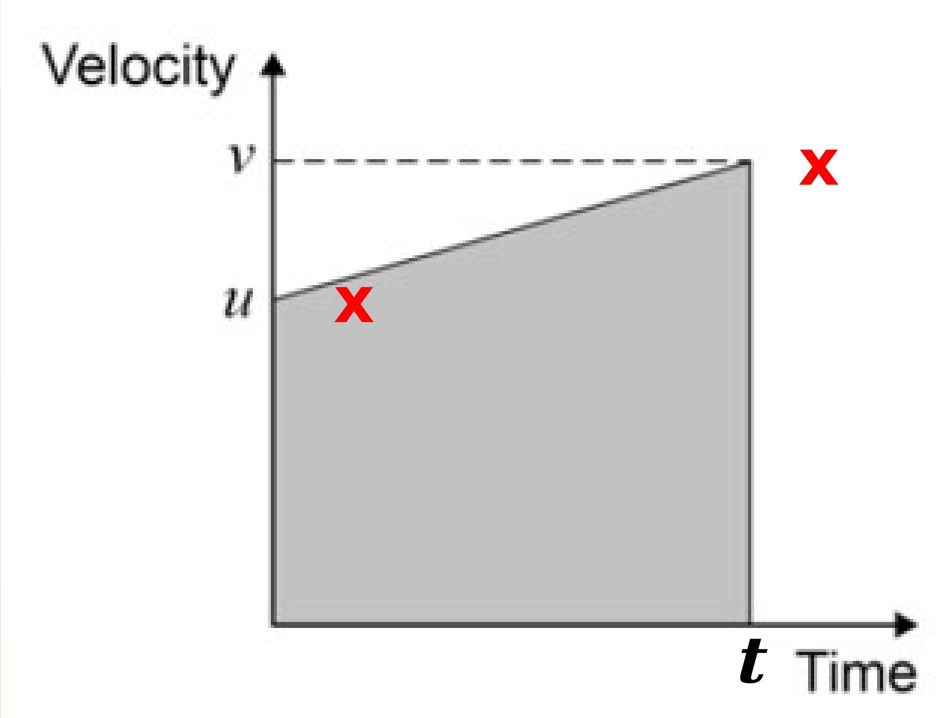
Displacement s (m)
Initial Velocity u (ms^{-1})
Final Velocity v (ms^{-1})
Acceleration a (ms^{-2})
Time t (s)

These five variables satisfy five equations (provided acceleration is ***constant***) known as the constant acceleration equations, equations of motion or SUVAT equations.

Each equation involves four variables, usually three would be known with the fourth to be

7.3 Equations of motion for constant acceleration

You must be able to derive these equations as follows...



- Label the starting velocity as
- Label the final velocity as
- Time is

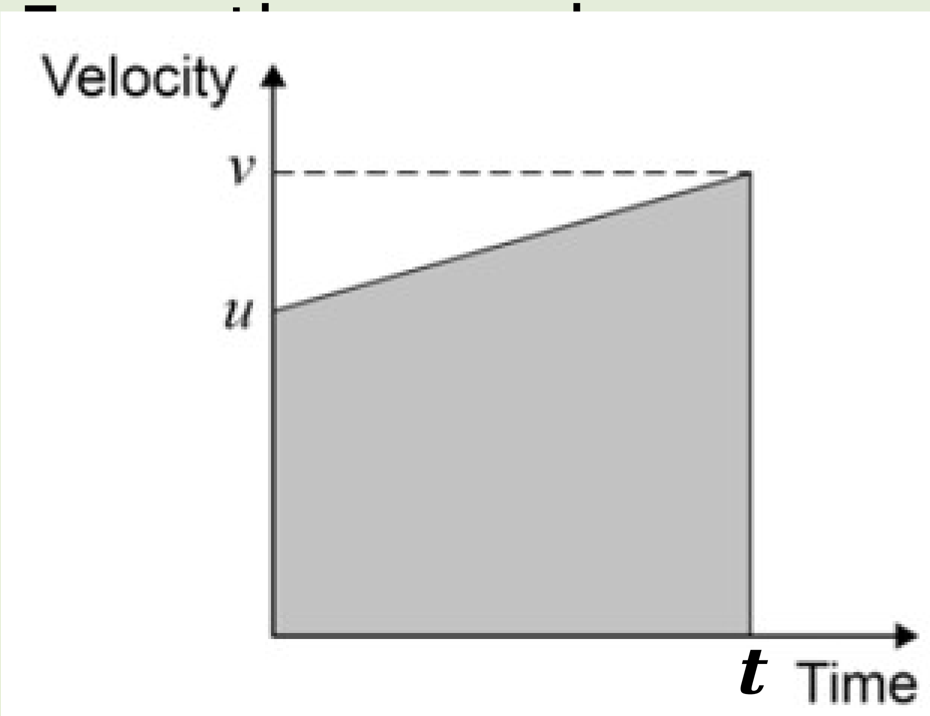
Acceleration = gradient

Rearranging so is the subject:

❶

7.3 Equations of motion for constant acceleration

You must be able to derive these equations as follows…



- From the area of a trapezium, the shaded area is

Area = displacement

7.3 Equations of motion for constant acceleration

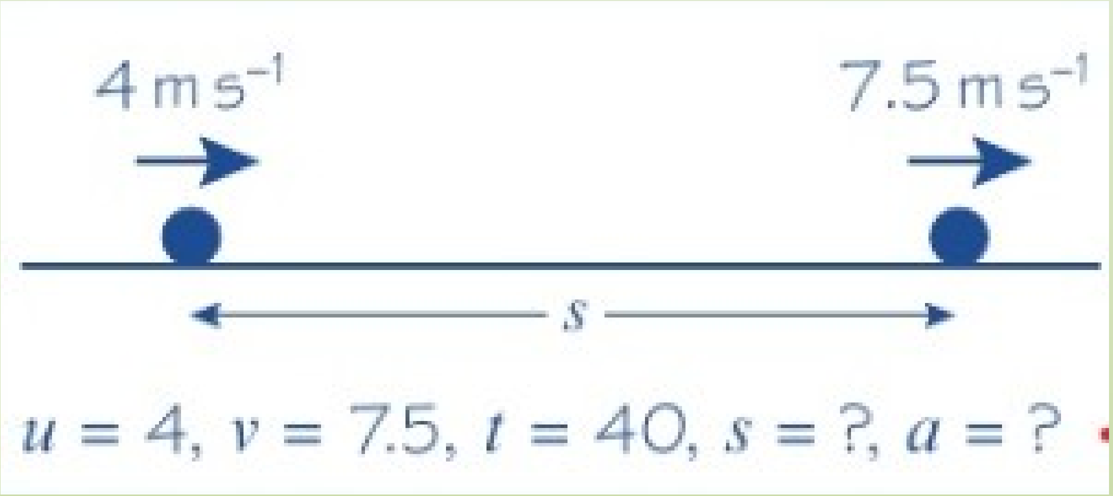
Example 1a

A cyclist is travelling along a straight road. She accelerates at a constant rate from a velocity of $4\ \mathrm{m\,s^{-1}}$ to a velocity of $7.5\ \mathrm{m\,s^{-1}}$ in 40 seconds. Find:

a the distance she travels in these 40 seconds

b her acceleration in these 40 seconds.

Using



7.3 Equations of motion for constant acceleration

Example 1b

A cyclist is travelling along a straight road. She accelerates at a constant rate from a velocity of 4 m s^{-1} to a velocity of 7.5 m s^{-1} in 40 seconds. Find:

a the distance she travels in these 40 seconds

b her acceleration in these 40 seconds.

Using

$u = 4,\ v = 7.5,\ t = 40,\ s = ?,\ a = ?$

7.3 Equations of motion for constant acceleration

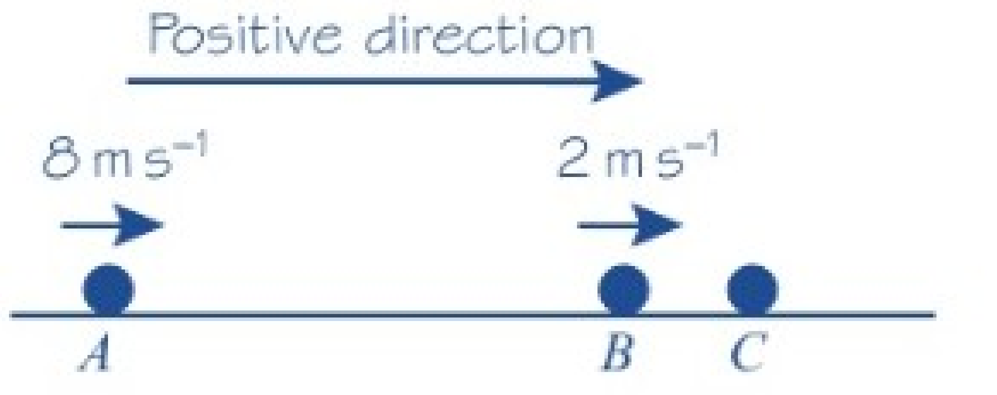
Example 2a

A particle moves in a straight line from a point A to a point B with constant deceleration $1.5\,\mathrm{m\,s^{-2}}$. The velocity of the particle at A is $8\,\mathrm{m\,s^{-1}}$ and the velocity of the particle at B is $2\,\mathrm{m\,s^{-1}}$. Find:

a the time taken for the particle to move from A to B

b the distance from A to B.

Using



$u = 8$, $v = 2$, $a = -1.5$, $t = ?$, $s = ?$

Acceleration is negative!

7.3 Equations of motion for constant acceleration

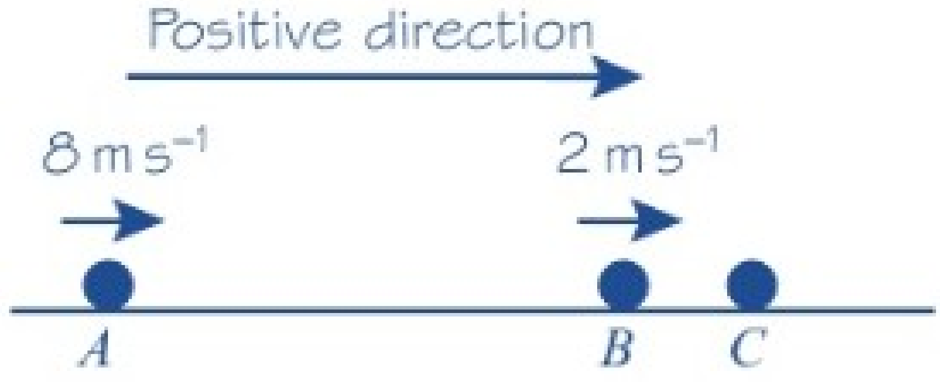
Example 2b

A particle moves in a straight line from a point A to a point B with constant deceleration $1.5\,\mathrm{m\,s^{-2}}$. The velocity of the particle at A is $8\,\mathrm{m\,s^{-1}}$ and the velocity of the particle at B is $2\,\mathrm{m\,s^{-1}}$. Find:

a the time taken for the particle to move from A to B

b the distance from A to B.

Using



$u = 8,\ v = 2,\ a = -1.5,\ t = ?,\ s = ?$

Use t = 4 from part (a)

7.3 Equations of motion for constant acceleration

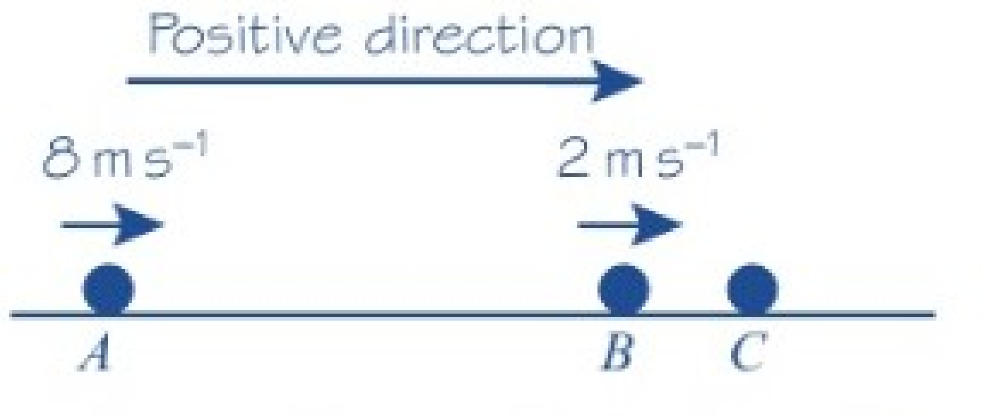
Example 2c

After reaching B the particle continues to move along the straight line with constant deceleration $1.5\,\mathrm{m\,s^{-2}}$. The particle is at the point C 6 seconds after passing through the point A. Find:

c the velocity of the particle at C

d the distance from A to C.

Using



$u = 8,\ v = 2,\ a = -1.5,\ t = ?,\ s = ?$

So it's moving back towards A!

Use t = 6

7.3 Equations of motion for constant acceleration

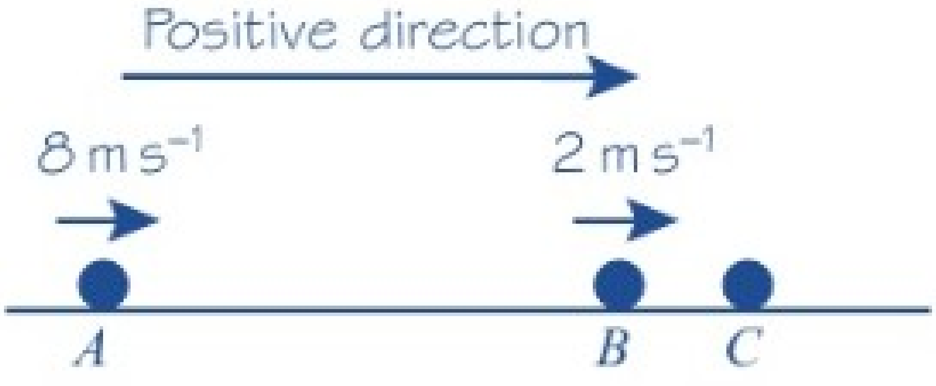
Example 2d

After reaching B the particle continues to move along the straight line with constant deceleration $1.5\,\text{m s}^{-2}$. The particle is at the point C 6 seconds after passing through the point A. Find:

c the velocity of the particle at C

d the distance from A to C.

Using



$u = 8,\ v = 2,\ a = -1.5,\ t = ?,\ s = ?$

Use t = 6

7.3 Equations of motion for constant acceleration

Example 3a

A car moves from traffic lights along a straight road with constant acceleration. The car starts from rest at the traffic lights and 30 seconds later the car passes a speed-trap where it is registered as travelling at 45 km h^{-1}. Find:

a the acceleration of the car

b the distance between the traffic lights and the speed-trap.

Using

$$45 \text{ km h}^{-1} = 45 \times \frac{1000}{3600} \text{ m s}^{-1} = 12.5 \text{ m s}^{-1}$$

$u = 0$, $v = 12.5$, $t = 30$, $a = ?$, $s = ?$

Convert 45km/h to m/s first

7.3 Equations of motion for constant acceleration

Example 3b

A car moves from traffic lights along a straight road with constant acceleration. The car starts from rest at the traffic lights and 30 seconds later the car passes a speed-trap where it is registered as travelling at $45\ \mathrm{km\,h^{-1}}$. Find:

a the acceleration of the car

b the distance between the traffic lights and the speed-trap.

Using

$$45\ \mathrm{km\,h^{-1}} = 45 \times \frac{1000}{3600}\ \mathrm{m\,s^{-1}} = 12.5\ \mathrm{m\,s^{-1}}$$

$u = 0,\ v = 12.5,\ t = 30,\ a = ?,\ s = ?$

Convert 45km/h to m/s first

7.3 Equations of motion for constant acceleration

From the graph, we derived the following equations:

From the gradient of the line:

❶ $$v = u + at$$

From the area under the graph:

❷ $$s = \frac{1}{2}\left(u + v\right)t$$

We can use these to derive three more formulae…

7.3 Equations of motion for constant acceleration

Substituting ❶ into ❷:

❶ $\square = \square + \square\square$

❷ $\square = \frac{1}{2}(\square + \square)\,\square$

❸

7.3 Equations of motion for constant acceleration

Rearranging ❶ for gives:

Substituting into ❷:

❶ $v = u + at$

❷ $s = \frac{1}{2}(u + v)t$

❸ $s = ut + \frac{1}{2}at^2$

❹

7.3 Equations of motion for constant acceleration

Rearranging ❶ for gives:

Substituting into ❷:

❶ $$v = u + at$$

❷ $$s = \frac{1}{2}\left(u + v\right)t$$

❸ $$s = ut + \frac{1}{2}at^2$$

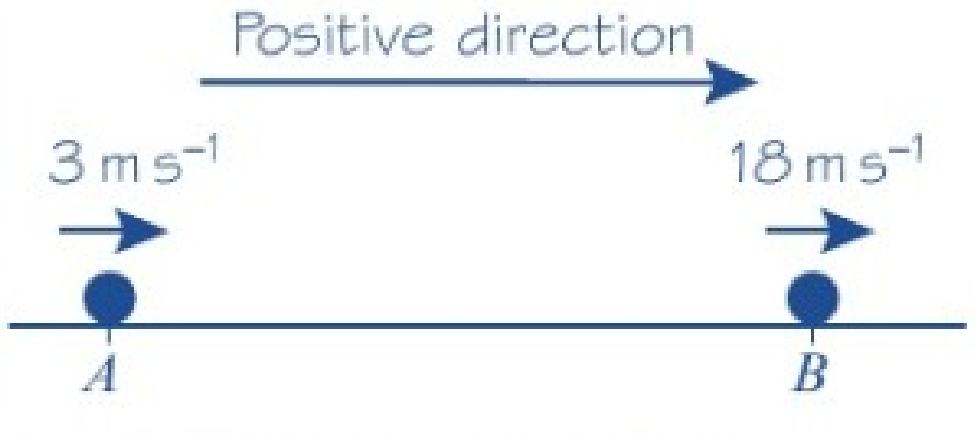
❹ $$s = vt - \frac{1}{2}at^2$$

❺

7.3 Equations of motion for constant acceleration

Example 4

A particle is moving along a straight line from A to B with constant acceleration $5\,\text{m}\,\text{s}^{-2}$. The velocity of the particle at A is $3\,\text{m}\,\text{s}^{-1}$ in the direction $\overrightarrow{AB}$. The velocity of the particle at B is $18\,\text{m}\,\text{s}^{-1}$ in the same direction. Find the distance from A to B.



s = s
u = 3ms^{-1}
v = 18ms^{-1}
a = 5ms^{-2}
t =

Using

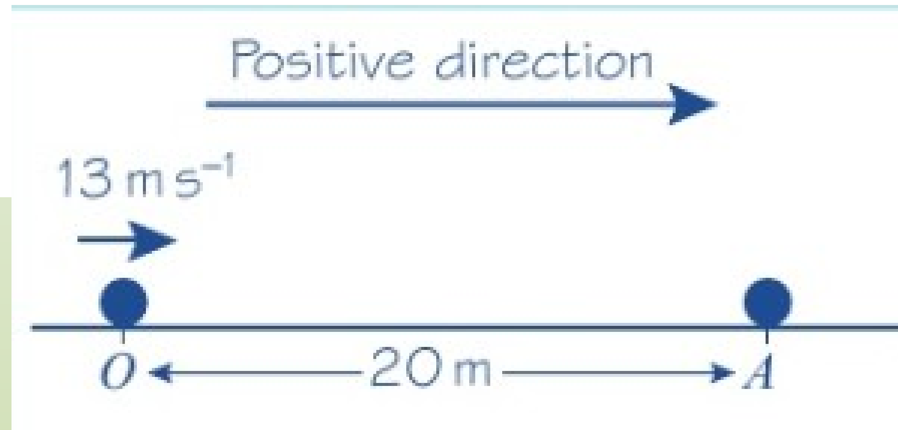
7.3 Equations of motion for constant acceleration

Example 5a

A particle is moving in a straight horizontal line with constant deceleration $4\,\mathrm{m\,s^{-2}}$. At time $t = 0$ the particle passes through a point O with speed $13\,\mathrm{m\,s^{-1}}$ travelling towards a point A, where $OA = 20\,\mathrm{m}$. Find:

a the times when the particle passes through A

b the value of t when the particle returns to O.



s = 20m

u = 13ms^{-1}

v =

a = -4ms^{-2}

Using

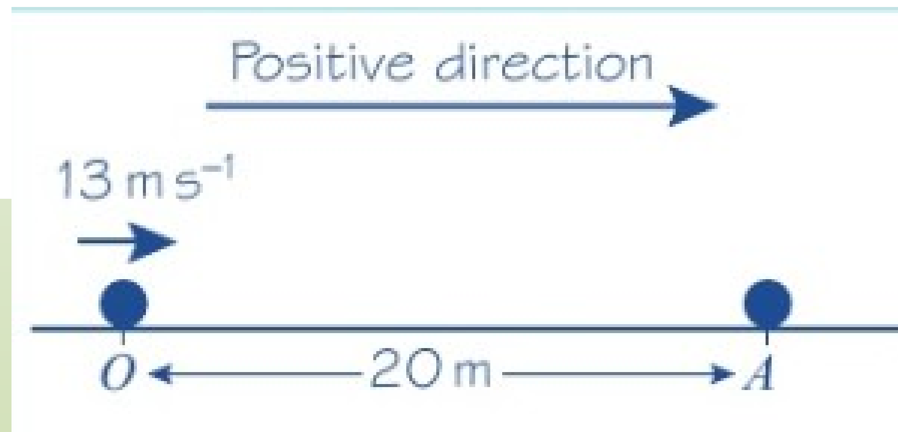
7.3 Equations of motion for constant acceleration

Example 5b

A particle is moving in a straight horizontal line with constant deceleration $4\,\text{m}\,\text{s}^{-2}$. At time $t = 0$ the particle passes through a point O with speed $13\,\text{m}\,\text{s}^{-1}$ travelling towards a point A, where $OA = 20\,\text{m}$. Find:

a the times when the particle passes through A

b the value of t when the particle returns to O.



s = 20m
u = 13ms^{-1}
v =
a = -4ms^{-2}

The particle returns to O when s=0

Using

The particle returns to O after 6.5 seconds

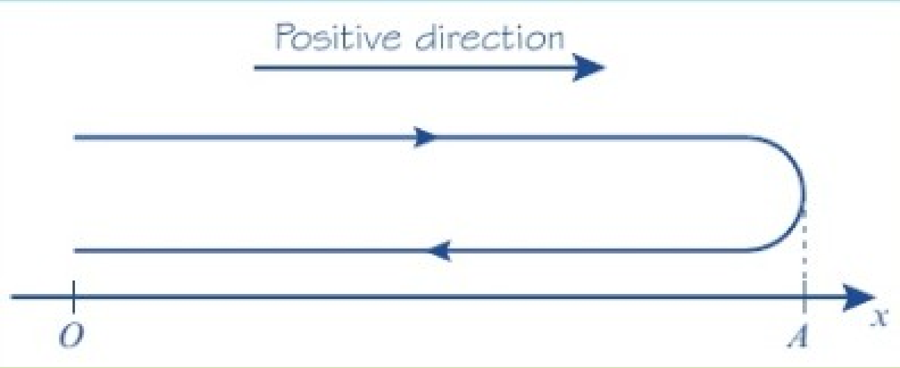
7.3 Equations of motion for constant acceleration

Example 6a

A particle P is moving on the x-axis with constant deceleration $2.5\,\mathrm{m\,s^{-2}}$. At time $t = 0$, the particle P passes through the origin O, moving in the positive direction of x with speed $15\,\mathrm{m\,s^{-1}}$. Find:

a the time between the instant when P first passes through O and the instant when it returns to O

b the total distance travelled by P during this time.



s = 0
u = 15ms^{-1}
v =
a = -2.5ms^{-2}
t = t

The particle returns to O when s=0

Using

The particle returns to O after 12 seconds

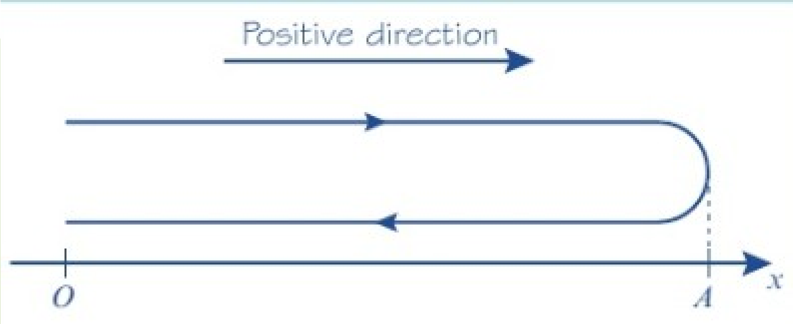
7.3 Equations of motion for constant acceleration

Example 6b

A particle P is moving on the x-axis with constant deceleration $2.5\,\text{m}\,\text{s}^{-2}$. At time $t = 0$, the particle P passes through the origin O, moving in the positive direction of x with speed $15\,\text{m}\,\text{s}^{-1}$. Find:

a the time between the instant when P first passes through O and the instant when it returns to O

b the total distance travelled by P during this time.



s = s
u = 15ms^{-1}
v = 0
a = -2.5ms^{-2}
t =

Before the particle changes direction, it has to come to a stop

Using

The total distance travelled is 2 x 45 = 90

7.3 Equations of motion for constant acceleration

Example 7

Car P is accelerating at $2ms^{-2}$. When its velocity is $10ms^{-1}$, it is overtaken by car Q, which is travelling at $16ms^{-1}$ and accelerating at $1ms^{-2}$. How long will P take to catch up with Q?

When P meets Q, their displacements must be the same at the same time (otherwise they don't meet)...

Car P:

$u_P = 10ms^{-1}$

$a_P = 2ms^{-2}$

Car Q:

$u_Q = 16ms^{-1}$

$a_Q = 1ms^{-2}$

Using Car P:

7.3 Equations of motion for constant acceleration

Example 7

Car P is accelerating at 2ms^{-2}. When its velocity is 10ms^{-1}, it is overtaken by car Q, which is travelling at 16ms^{-1} and accelerating at 1ms^{-2}. How long will P take to catch up with Q?

When P meets Q, their displacements must be the same at the same time (otherwise they don't meet)…

Car P:

u_P = 10ms^{-1}

a_P = 2ms^{-2}

Car Q:

u_Q = 16ms^{-1}

a_Q = 1ms^{-2}

Using Car Q:

7.3 Equations of motion for constant acceleration

Example 7

When P meets Q, $s_P = s_Q$:

Hence, when t = 0s, this is at the start when Q passes P, so P catches Q when t = 12s.